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AN IMPROVED MIXING LENGTH THEORY OF TURBULENT HEAT AND MASS TRANSFER

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Abstract—An improved mixing length theory of turbulent heat and mass transfer is developed which applies more realistically when the velocity gradient, or the temperature gradient, or both, are small. The theory is applied to turbulent flow between parallel plates which are maintained at constant but different temperatures, and the results compare favorably with experimental data.

NOMENCLATURE

A, constant $= 5{\cdot}4/k$;
B, constant $= 5{\cdot}5$;
C, $c_1 c_2$;
c_1, defined in equation (7);
c_2, velocity correlation coefficient;
c_3, temperature correlation coefficient, equation (8);
c_H, $c_1 c_3$;
c_p, specific heat of the fluid;
G, dimensionless temperature gradient at $y^+ = 0$;
K, thermal conductivity of the fluid;
k, constant $= 0{\cdot}4$;
l, mixing length;
l_1, lu^*/ν;
l_p, $\sqrt{(c_H)}\, l_1$;
Pr, $c_p \mu/K$, Prandtl number;
Pr_t, $\varepsilon_M/\varepsilon_H$, turbulent Prandtl number;
Re, $4 y_m u_b/\nu$, Reynolds number;
$\bar{T}$, time average temperature;
T', fluctuating component of temperature;
T_{w_1}, T_{w_2}, temperature of the lower and upper walls, see Fig. 1;
T_c, temperature at the centerline;
T_i, inlet temperature, see Fig. 1;
ΔT, $T_i - T_w$;
$\bar{u}$, time average axial velocity;
u', fluctuating component of the velocity;
u_b, area average velocity;
u^*, $\sqrt{(\tau_w/\rho)}$, friction velocity;
u^+, $\bar{u}/u^*$;
v', fluctuating component of the velocity in the y direction;
y, transverse coordinate measured from the wall, see Fig. 1;
y^+, yu^*/ν;
y_m, half the distance between the two plates, see Fig. 1;
y_m^+, $y_m u^*/\nu$;
ε_M, eddy diffusion coefficient for momentum transport;
ε_H, eddy diffusion coefficient for heat transport;
θ, $(\bar{T} - T_{w_1})/(T_i - T_{w_1})$;
θ_w, $(T_{w_2} - T_{w_1})/(T_i - T_{w_1})$;
μ, molecular viscosity of the fluid;
ν, μ/ρ, kinematic viscosity of the fluid;
ρ, density of the fluid;
τ_w, shear stress at the wall.

RECENTLY it was shown [1, 2] how Prandtl's mixing length theory for momentum transport can be extended in a simple manner, so that it is applicable in regions where the velocity gradient is very small as it is in the central core of fully developed turbulent tube flow. According to this more general phenomenological theory, which was shown to include Prandtl's momentum transport and von Karman's similarity hypotheses as special cases, the Reynolds stress can be written as

$$-\frac{\overline{u'v'}}{u^{*2}} = \begin{cases} c l_1^2 \left(\frac{du^+}{dy^+}\right)^2, & \left|l_1 \frac{du^+}{dy^+}\right| \geqslant \left|\frac{l_1^2}{2}\frac{d^2u^+}{dy^{+2}}\right| \quad (1a) \\ \frac{c}{4} l_1^4 \left(\frac{d^2u^+}{dy^{+2}}\right)^2, & \left|\frac{l_1^2}{2}\frac{d^2u^+}{dy^{+2}}\right| \geqslant \left|l_1 \frac{du^+}{dy^+}\right|. \quad (1b) \end{cases}$$

Equation (1a) is the result obtained by Prandtl whereas equation (1b) is a new result which is to be used in the region where the velocity gradient is very small. Our purpose here is to improve the mixing length theory of turbulent heat and mass transport processes in a similar manner so that it is applicable equally well in regions where velocity, temperature or concentration gradients are both large and small.

In this discussion we will concentrate our attention on heat transfer only since appropriate data are available for comparison with the theory. Equations for mass transfer can be obtained in exactly the same fashion except when mass transfer rates are high enough for transverse convection to become important [3,4].

Mixing length theory assumes that the fluid lumps retain their identity over a certain distance and then mix with the surroundings. Thus, when a mass of fluid traverses a distance l in the positive or negative y direction, the corresponding change in temperature is given by

$$|\Delta T_1| = |\overline{T}(y) - \overline{T}(y - l)| \approx \left|l\frac{\partial \overline{T}}{\partial y} - \frac{l^2}{2}\frac{\partial^2 \overline{T}}{\partial y^2}\right| \quad (2)$$

or

$$|\Delta T_2| = |\overline{T}(y + l) - \overline{T}(y)| \approx \left|l\frac{\partial \overline{T}}{\partial y} + \frac{l^2}{2}\frac{\partial^2 \overline{T}}{\partial y^2}\right|. \quad (3)$$

In equations (2) and (3) the temperature $\overline{T}(y)$ is expanded in a Taylor series in the positive and negative directions, retaining terms containing second order derivatives.

Let us now assume that the time-average of the absolute value of the fluctuation caused by the temperature differences ΔT_1 and ΔT_2 can be written as,

$$\overline{|T'|} = \frac{1}{2}(|\Delta T_1| + |\Delta T_2|) = \frac{1}{2}\left[\left|l\frac{\partial \overline{T}}{\partial y} - \frac{l^2}{2}\frac{\partial^2 \overline{T}}{\partial y^2}\right| + \left|l\frac{\partial \overline{T}}{\partial y} + \frac{l^2}{2}\frac{\partial^2 \overline{T}}{\partial y^2}\right|\right] \quad (4)$$

or in terms of dimensionless quantities we have

$$\frac{\overline{|T'|}}{\Delta T} = \frac{1}{2}\left[\left|l_1\frac{\partial \theta}{\partial y^+} - \frac{l_1^2}{2}\frac{\partial^2 \theta}{\partial y^{+2}}\right| + \left|l_1\frac{\partial \theta}{\partial y^+} + \frac{l_1^2}{2}\frac{\partial^2 \theta}{\partial y^{+2}}\right|\right]. \quad (5)$$

By following a similar procedure, for the velocity fluctuation one obtains [1, 2],

$$\frac{\overline{|u'|}}{u^*} = \frac{1}{2}\left[\left|l_1\frac{du^+}{dy^+} - \frac{l_1^2}{2}\frac{d^2u^+}{dy^{+2}}\right| + \left|l_1\frac{du^+}{dy^+} + \frac{l_1^2}{2}\frac{d^2u^+}{dy^{+2}}\right|\right]. \quad (6)$$

We assume with Prandtl that the transverse component v' is proportional to u', so that

$$\overline{|v'|} = c_1 \overline{|u'|}. \tag{7}$$

Also, the temperature–velocity correlation coefficient c_3 is defined as

$$c_3 = -\frac{\overline{T'v'}}{\overline{|T'|}\,\overline{|v'|}}. \tag{8}$$

By combining equations (5)–(8) we get

$$-\frac{\overline{T'v'}}{\Delta T u^*} = \frac{c_H}{4}\left[\left|l_1\frac{\mathrm{d}u^+}{\mathrm{d}y^+} - \frac{l_1^2}{2}\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right| + \left|l_1\frac{\mathrm{d}u^+}{\mathrm{d}y^+} + \frac{l_1^2}{2}\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right|\right]\left[\left|l_1\frac{\partial\theta}{\partial y^+} - \frac{l_1^2}{2}\frac{\partial^2\theta}{\partial y^{+2}}\right| + \left|l_1\frac{\partial\theta}{\partial y^+} + \frac{l_1^2}{2}\frac{\partial^2\theta}{\partial y^{+2}}\right|\right] \tag{9}$$

where

$$c_H = c_1 c_3. \tag{10}$$

Since

$$\tfrac{1}{2}[|a-b| + |a+b|] = \begin{cases} |a| \;, & |a| \geqslant |b| \\ |b| \;, & |b| \geqslant |a|. \end{cases} \tag{11}$$

Equation (9) reduces to

$$-\frac{\overline{T'v'}}{\Delta T u^*} = \begin{cases} c_H l_1^2 \left|\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right|\left|\frac{\partial\theta}{\partial y^+}\right| \;; & \left|l_1\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right| \geqslant \left|\frac{l_1^2}{2}\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right|,\ \left|l_1\frac{\partial\theta}{\partial y^+}\right| \geqslant \left|\frac{l_1^2}{2}\frac{\partial^2\theta}{\partial y^{+2}}\right| & \text{(12a)} \\ \frac{c_H}{2} l_1^3 \left|\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right|\left|\frac{\partial\theta}{\partial y^+}\right| \;; & \left|\frac{l_1^2}{2}\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right| \geqslant \left|l_1\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right|,\ \left|l_1\frac{\partial\theta}{\partial y^+}\right| \geqslant \left|\frac{l_1^2}{2}\frac{\partial^2\theta}{\partial y^{+2}}\right| & \text{(12b)} \\ \frac{c_H}{2} l_1^3 \left|\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right|\left|\frac{\partial^2\theta}{\partial y^{+2}}\right| \;; & \left|l_1\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right| \geqslant \left|\frac{l_1^2}{2}\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right|,\ \left|\frac{l_1^2}{2}\frac{\partial^2\theta}{\partial y^{+2}}\right| \geqslant \left|l_1\frac{\partial\theta}{\partial y^+}\right| & \text{(12c)} \\ \frac{c_H}{4} l_1^4 \left|\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right|\left|\frac{\partial^2\theta}{\partial y^{+2}}\right| \;; & \left|\frac{l_1^2}{2}\frac{\mathrm{d}^2u^+}{\mathrm{d}y^{+2}}\right| \geqslant \left|l_1\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right|,\ \left|\frac{l_1^2}{2}\frac{\partial^2\theta}{\partial y^{+2}}\right| \geqslant \left|l_1\frac{\partial\theta}{\partial y^+}\right| & \text{(12d)} \end{cases}$$

Equation (12a) is the familar result obtained by the usual extension of Prandtl's momentum transfer mixing length theory to heat transfer problems [3]. Equations (12b, c, d), on the other hand, arise because of the inclusion of the second order derivatives of velocity and temperature in the Taylor series expansion, equations (2) and (3). Thus, equations (12b, c, d) will be useful in regions where the velocity and/or temperature gradient is small as, for example, in the following four cases:

(a) Fully developed turbulent flow between parallel plates with constant heat flux at the walls.

In a region near the wall both the velocity and the temperature gradients are large and therefore, equation (12a) will apply, whereas in the central region, because of symmetry, these gradients are small and consequently, equation (12d) applies.

(b) Fully developed turbulent flow between parallel plates with walls maintained at constant but different temperatures.

Equation (12a) applies in a region near the wall. In the central portion, the velocity gradient is small because of symmetry, and the temperature gradient is still significant so that equation (12b) applies.

(c) Couette flow between two parallel walls displaced relative to each other with two walls maintained at the same constant temperature.

In this case equation (12a) again applies near the walls. Near the central region, the velocity gradient is significant but the temperature gradient is small, because of symmetry, and therefore equation (12c) will apply.

(d) Turbulent heat and mass transfer across a liquid film.

In this case a variety of possibilities exist depending on whether or not there is shear at the free surface. In the simplest case of no shear at the gas–liquid interface, equation (12a) applies near the wall and equation (12b) near the free surface if there is heat or mass flux at the wall. If the wall is adiabatic or impervious to mass transfer, then equation (12c) rather than (12a) applies near the wall.

It should be emphasized that the mixing length theory is inadequate to give clear physical insight into the structure of turbulent flow because no attempt is made to explain how and why a fluid lump retains its identity and the mechanism by which it adopts the properties of the surrounding. Nevertheless, it is believed that equation (12) is a useful extension for the purpose of making heat transfer calculations. To illustrate this point consider the case (b), mentioned above, of asymmetric heat transfer between parallel plates. This is a physical situation for which the mathematical formulation is particularly simple, Prandtl's theory is clearly inadequate and experimental data are available to compare with the theory.

By assuming constant physical properties, the fully developed temperature profile at large distances downstream from the start of the heat transfer section can be obtained from (see Fig. 1)

$$\frac{d}{dy^+}\left[\frac{1}{Pr}\frac{d\theta}{dy^+} - \frac{\overline{T'v'}}{\Delta T u^*}\right] = 0 \tag{13}$$

$$\theta(0) = 0 \tag{14}$$

$$\theta(2y_m^+) = \theta_w. \tag{15}$$

Now for $-\overline{T'v'}$ we will substitute appropriate expressions from equation (12). Since equation

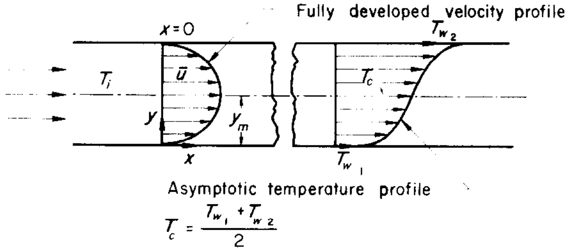

FIG. 1. Schematic diagram of parallel plate duct with unequal wall temperatures.

(12) contains velocity gradients, it is necessary to solve first the momentum equations. This has been done [1, 2] by dividing the region between wall and centerline into four regions. The momentum eddy diffusivity obtained for each region and the assumptions made for each region to solve the heat transfer problem are given below

(1) The laminar sublayer: $0 \leqslant y^+ \leqslant 5$.

In this region

$$\frac{1}{Pr}\frac{d\theta}{dy^+} \gg -\frac{\overline{T'v'}}{\Delta T u^*}.$$

(2) The buffer layer: $5 \leqslant y^+ \leqslant 30$.

Let us express $-\overline{T'v'}$ in terms of the eddy diffusivity ε_H,

$$-\frac{\overline{T'v'}}{\Delta T u^*} = \frac{\varepsilon_H}{\nu}\frac{d\theta}{dy^+} = \frac{1}{Pr_t}\frac{\varepsilon_M}{\nu}\frac{d\theta}{dy^+}$$

where eddy diffusivity for momentum transfer ε_M, is defined as

$$-\frac{\overline{u'v'}}{u^{*2}} = \frac{\varepsilon_M}{\nu}\frac{du^+}{dy^+}.$$

In general, Pr_t is a function of y^+ but it apparently does not play a very important role when the Prandtl number is close to unity or higher.

Thus, for present purposes we will assume that $Pr_t = 1$,

$$-\frac{\overline{T'v'}}{\Delta T u^*} = \frac{\varepsilon_M}{\nu}\frac{d\theta}{dy^+} = \left(\frac{y^+}{5} - 1\right)\frac{d\theta}{dy^+}.$$

(3) The inner layer: $30 \leqslant y^+ \leqslant 0{\cdot}4\, y_m^+$.

In this region heat transferred by molecular conduction can be neglected compared to the transport by eddies. Consequently,

$$-\frac{\overline{T'v'}}{\Delta T u^*} \gg \frac{1}{Pr}\frac{d\theta}{dy^+}.$$

Since both the velocity and the temperature gradients are large, equation (12a) applies

$$-\frac{\overline{T'v'}}{\Delta T u^*} = c_H l_1^2 \left(\frac{du^+}{dy^+}\right)\left(\frac{d\theta}{dy^+}\right).$$

By assuming that $Pr_t = 1$, we have [1, 2]

$$\frac{\varepsilon_M}{\nu} = c_H l_1^2 \left(\frac{du^+}{dy^+}\right) = k y^+ \left(1 - \frac{y^+}{y_m^+}\right)^{\frac{1}{2}}.$$

(4) The outer layer: $0{\cdot}4\, y_m^+ \leqslant y^+ \leqslant y_m^+$.
Again,

$$-\frac{\overline{T_2'v'}}{\Delta T u^*} \gg \frac{1}{Pr}\frac{d\theta}{dy^+}.$$

In this region the velocity gradient is small but the temperature gradient is significant compared to the corresponding second derivatives so that equation (12b) applies:

$$-\frac{\overline{T_2'v'}}{\Delta T u^*} = \frac{c_H}{2} l_1^3 \left|\frac{d^2u^+}{dy^{+2}}\right| \frac{d\theta}{dy^+} = \frac{\varepsilon_M}{\nu}\frac{d\theta}{dy^+}$$

and [1, 2]

$$\frac{\varepsilon_M}{\nu} = \frac{y_m^+}{A} = \frac{k y_m^+}{5{\cdot}4}.$$

Division of the duct into the four regions employed here will give useful results only for moderate Prandtl numbers ($Pr \simeq 0{\cdot}5$–20). At very high Prandtl numbers the concept of a "laminar" sublayer does not apply. Since eddies do penetrate this region the contribution to heat transfer by turbulent motion cannot be neglected when $Pr \gg 1$, as can be seen from equation (13). That is, when molecular diffusion is very slow, even small amounts of eddy diffusion contribute significantly to the transport process. On the other hand when $Pr \ll 1$, molecular conduction in the radial direction [see equation (13)] cannot be neglected in the central portion of the duct, as we have done here.

Thus, for moderate Prandtl numbers one can divide the duct into four regions and with the above assumptions equation (13) can be solved to obtain the temperature profile [2]

$$\theta = G\,.\,y^+, \qquad 0 \leqslant y^+ \leqslant 5 \qquad (16)$$

$$\theta = \frac{5G}{Pr}\left\{Pr + \ln\left[1 + Pr\left(\frac{y^+}{5} - 1\right)\right]\right\}, \qquad 5 \leqslant y^+ \leqslant 30 \qquad (17)$$

$$\theta = \frac{5G}{Pr}\{Pr + \ln(1 + 5Pr)\} + \frac{G}{Pr}\frac{1}{k}\left\{\ln\left[\frac{1 - \sqrt{[1 - (y^+/y_m^+)]}}{1 + \sqrt{[1 - (y^+/y_m^+)]}}\,\frac{1 + \sqrt{[1 - (30/y_m^+)]}}{1 - \sqrt{[1 - (30/y_m^+)]}}\right] + \frac{2}{\sqrt{[1 - y^+/y_m^+)]}} - \frac{2}{\sqrt{[1 - (30/y_m^+)]}}\right\}, \qquad 30 \leqslant y^+ \leqslant 0{\cdot}4\, y_m^+ \qquad (18)$$

$$\theta = \frac{5G}{Pr}\{Pr + \ln(1 + 5Pr)\} + \frac{G}{Pr}\frac{1}{k}\left\{\ln\left[\frac{1 - \sqrt{(0{\cdot}6)}}{1 + \sqrt{(0{\cdot}6)}}\,\frac{1 + \sqrt{[1 - (30/y_m^+)]}}{1 - \sqrt{[1 - (30/y_m^+)]}}\right] + \frac{2}{\sqrt{(0{\cdot}6)}} - \frac{2}{\sqrt{[1 - (30/y_m^+)]}}\right\} + \frac{G}{Pr}\frac{5{\cdot}4}{k}\left(\frac{y^+}{y_m^+} - 0{\cdot}4\right) \qquad (19)$$

$$0{\cdot}4 y_m^+ \leqslant y^+ \leqslant y_m^+$$

where G is the gradient at the wall and it is obtained by employing the relation $\theta(y_m^+) = \theta_w/2$. This gives

$$\frac{Pr}{2}\frac{\theta_w}{G} = 5[Pr + \ln(1 + 5Pr)] + \frac{1}{k} \times \left\{\ln\left[\frac{1 - \sqrt{(0{\cdot}6)}}{1 + \sqrt{(0{\cdot}6)}}\,\frac{1 + \sqrt{[1 - (30/y_m^+)]}}{1 - \sqrt{[1 - (30/y_m^+)]}}\right]\right.$$

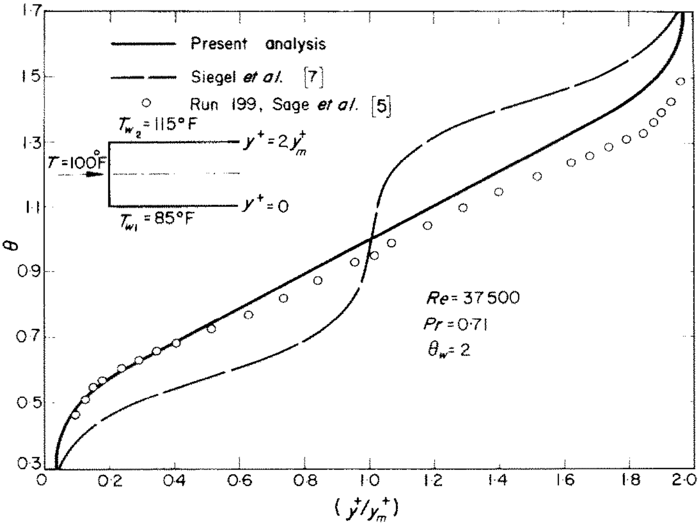

FIG. 2. Comparison of theoretical fully developed temperature profile with experimental data. (The dashed curve is taken from Blanco [8]. The eddy diffusivity expression proposed by Siegel *et al.* [7] was used to obtain this curve.)

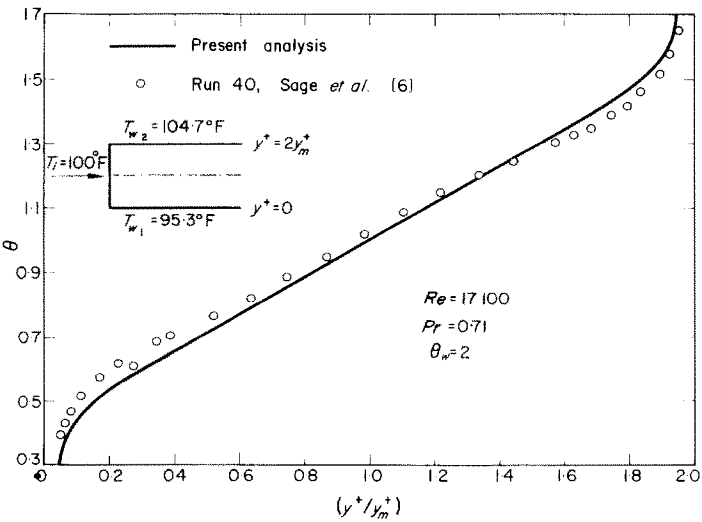

FIG. 3. Comparison of theoretical fully developed temperature profile with experimental data.

$$+\frac{2}{\sqrt{(0{\cdot}6)}}-\frac{2}{\sqrt{[1-(30/y_m^+)]}}+3{\cdot}24\bigg\}. \quad (20)$$

In equation (19) k is the constant which is determined from the velocity profile measurements. If $k = 0{\cdot}4$ and $B = 5{\cdot}5$, the relationship between the Reynolds number and y_m^+ can be obtained by integrating the velocity profile [2] and this gives

$$(Re/4) = 2{\cdot}5\, y_m^+ \ln(1{\cdot}35\, y_m^+) + 2{\cdot}9\, y_m^+ - 64{\cdot}5. \quad (21)$$

Thus, the temperature profile is completely determined when the parameters Pr, Re and θ_w are specified.

Temperature profiles obtained from these equations are shown in Figs. 2 and 3. The results are compared with the experimental data of Sage *et al.* [5, 6] and the agreement is quite satisfactory, particularly in the central portion,

$$0{\cdot}4 \leqslant \frac{y^+}{y_m^+} \leqslant 1{\cdot}6.$$

It is in this region that we have used the modified form of the mixing length theory, equation (12b). The original form of Prandtl's mixing length theory gave the dashed temperature profile [7, 8] which obviously does not agree with the experimental data. Therefore, it appears that the modification of the mixing length theory proposed here is useful and is consistent with experimental observations. Some error may have been introduced by neglecting dissipation and expansion work, especially for the run 199 in Fig. 2 [9].

Let us now examine the essential difference between the present theory and previous work. This difference arises mainly because different mixing length expressions have been employed in the central portion of the channel.

In the central portion the contribution to heat transfer by turbulence is much greater than that due to molecular motion. Therefore, equation (13) after integration can be written as

$$-\frac{\overline{T'v'}}{\Delta T u^*} \simeq \frac{1}{Pr}\left(\frac{\mathrm{d}\theta}{\mathrm{d}y^+}\right)_{y^+=0}, \quad 0{\cdot}4 \leqslant \frac{y^+}{y_m^+} \leqslant 1{\cdot}6.$$

If one employs Prandtl's mixing length theory to obtain $-\overline{T'_2v'}$, i.e. equation (12a), the result is

$$-\frac{\overline{T'v'}}{\Delta T u^*} = l_p^2\left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)\left(\frac{\mathrm{d}\theta}{dy^+}\right) \simeq \frac{1}{Pr}\left(\frac{\mathrm{d}\theta}{\mathrm{d}y^+}\right)_{y^+=0}.$$

or

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}y^+}\right) = \frac{1}{Pr}\left(\frac{\mathrm{d}\theta}{\mathrm{d}y^+}\right)_{y_+=0}\left[\frac{1}{l_p^2(\mathrm{d}u^+/\mathrm{d}y^+)}\right]$$

where $l_p = l_1\sqrt{(c_H)}$. Since the velocity gradient is zero at the centreline, if we take l_p to be finite, the above equation predicts that the temperature gradient at the centerline is infinity (see Fig. 2),

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}y^+}\right)_{y^+=y^+_{\max}} = \infty.$$

This is obviously contrary to experimental observations as can be seen clearly from the figures. In some of the problems studied, such as momentum and heat transfer in a tube, because of symmetry, the velocity and temperature gradients at the center are zero. However, Prandtl's mixing length theory cannot account for both a finite eddy diffusivity and zero gradients simultaneously without being modified arbitrarily to do so and therefore it is considered to be inconsistent in the central portion of the tube. Nevertheless, the velocity and temperature at the centerline, predicted from the mixing length theory, are found to be in reasonable agreement with the experimental values. Thus, the Prandtl and von Karman analogies give fairly satisfactory results for $0{\cdot}5 < Pr < 20$. In other problems involving asymmetric condition, similar to the one we have considered here, in the past mixing length theory has been modified in some arbitrary but simple way to overcome the difficulty of an infinite gradient at the center. For example, mixing length theory is found to give satisfactory results when the eddy diffusivity curve is arbitrarily flattened from the point of its maximum to the centerline [8, 9]. Such a condition is also important when one is concerned with interphase transfer in multiphase cocurrent or countercurrent flow [10]. The improved mixing length theory proposed here, essentially does the same thing but in a more systematic way on the basis of our observations and knowledge of turbulence. No arbitrary modifications are necessary and, in particular, it answers clearly two main questions: (1) Why is Prandtl's mixing length theory not applicable in the central core of turbulent flow in conduits? and (2) What is a simple systematic way to overcome this difficulty? The reformulation of Prandtl's mixing length theory, which includes the $(d^2\bar{u}/dy^2)$ and $(d^2\theta/dy^{+2})$ terms in equations (5) and (6), gives answers to both of these questions.

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UNE THÉORIE AMÉLIORÉE DE LA LONGUEUR DE MÉLANGE POUR LE TRANSFERT THERMIQUE ET MASSIQUE TURBULENT

Résumé—On développe une théorie améliorée de la longueur de mélange pour le transfert thermique et massique laquelle s'applique avec plus de pertinence lorsque le gradient de vitesse ou le gradient de température ou encore les deux à la fois sont faibles. La théorie est appliquée à l'écoulement turbulent entre deux plans parallèles maintenus à des températures constantes et différentes. Les résultats sont comparés favorablement avec les résultats expérimentaux.

EINE VERBESSERTE MISCHLÄNGENTHEORIE

Zusammenfassung—Eine verbesserte Mischlängentheorie für den turbulenten Wärme- und Stoffübergang wurde entwickelt. Sie gewährleistet realistischere Anwendbarkeit, wenn der Geschwindigkeitsgradient oder Temperaturgradient oder beide klein sind. Die Theorie wird auf turbulente Strömung zwischen parallelen Platten konstanter, aber ungleicher Temperatur angewandt, und die Ergebnisse lassen sich gut mit Versuchswerten vergleichen.

УСОВЕРШЕНСТВОВАННАЯ ТЕОРИЯ ПУТИ СМЕШЕНИЯ ДЛЯ ТУРБУЛЕНТНОГО ТЕПЛО-И МАССООБМЕНА

Аннотация—Разработана усовершенствованная теория пути смешения для тепло-и массопереноса, наиболее надёжная при малых градиентах скорости и температуры. Эта теория применена к турбулентному течению между параллельными пластинами, которые поддерживаются при постоянных, но разных температурах. Результаты сравнения с экспериментальными данными являются удовлетворительными.